# Illustrative view on the role of spin-orbit coupling in magnetocrystalline anisotropy of adatoms 

O. Šipr ${ }^{1}$<br>S. Mankovsky ${ }^{2}$<br>S. Bornemann ${ }^{2}$ J. Minár ${ }^{2,3}$<br>H. Ebert ${ }^{2}$<br>${ }^{1}$ Institute of Physics AS CR, Prague http://www.fzu.cz/~sipr<br>${ }^{2}$ Department Chemie, Ludwig-Maximilians-Universität München<br>${ }^{3}$ New Technologies Research Center, University of West Bohemia, Pilsen

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## Outline

- MAE: What is it about?
- SOC-induced splitting of DOS:

How to do it

- SOC-induced splitting of DOS:

Ab-initio results

- SOC-induced splitting of DOS:

Model Hamiltonian results

- Summary


## Magnetic Anisotropy Energy（MAE）

－Difference between total energies for two orientations of the magnetization $\hat{M}$ with respect to the crystal lattice：

$$
E_{\mathrm{MAE}}=E\left(\hat{\mathbf{M}}_{1}\right)-E\left(\hat{\mathbf{M}}_{2}\right)
$$

－Magnetocrystalline contribution to MAE is due to the spin orbit coupling（SOC）．

## Magnetic Anisotropy Energy (MAE)

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- Magnetocrystalline contribution to MAE is due to the spin orbit coupling (SOC).
- What is the specific mechanism that gives rise to the MAE for a concrete system?
- Can we see a signature of MAE in the electronic structure?
- People strive to understand MAE to make it as high as possible [Jesche et al. Nature Comm. (2014), Rau et al. Science (2014), Khajetoorians \& Wiebe Science (2014), Antropov \& Antonov PRB (2014), Antropov et al. SSC (2014)].


## Total energies and single-particle (band) energies

- Total energy within the DFT framework:

$$
E_{\mathrm{tot}}=\sum_{i}^{\mathrm{occ}} E_{i}-\frac{1}{2} \int \mathrm{~d} \mathbf{r} \int \mathrm{~d} \mathbf{r}^{\prime} \frac{n(\mathbf{r}) n\left(\mathbf{r}^{\prime}\right)}{\left|\mathbf{r}-\mathbf{r}^{\prime}\right|}+E_{\mathrm{xc}}[n]-\int \mathrm{d} \mathbf{r} n(\mathbf{r}) v_{\mathrm{xc}}[n](\mathbf{r})
$$

- Kohn-Sham functional is extremal $\Rightarrow$ differences in $E_{\text {tot }}$ can be approximated by differences in single-particle or "band" energies:

$$
E_{\mathrm{MAE}} \approx \sum_{i}^{\circ \circ \mathrm{Cc}} E_{i}\left(\hat{\mathbf{M}}_{1}\right)-\sum_{i}^{\text {occ }} E_{i}\left(\hat{\mathbf{M}}_{2}\right)
$$

Issue to be addressed:
How are the band energies affected by the spin-orbit coupling?

## Change of band energy upon inclusion of SOC

- SOC-perturbed Hamiltonian:

$$
H=H_{0}+H_{\mathrm{SOC}}=H_{0}+\xi(r) \sigma \cdot \mathbf{L}
$$

- Lowest-order non-vanishing correction to the ground state is the second-order term:

$$
\delta E^{(2)}=\sum_{j \neq 0} \frac{\left.\left|\left\langle\psi_{0}\right| H_{\mathrm{SOC}}\right| \psi_{j}\right\rangle\left.\right|^{2}}{E_{0}-E_{j}}
$$

However:
There may be contributions to the MAE which cannot be described within perturbation theory (e.g., if SOC is large or if degenerate states are important).

## If the perturbation theory cannot be used. . .

- Inspiration:

For free atoms, SOC splits the originally degenerate states by $\sim m_{\ell} \lambda \cos \theta$,
$m_{\ell}$ is the (orbital) magnetic quantum number, $\lambda$ is the SOC scaling factor,
$\theta$ is the angle between $\hat{\boldsymbol{M}}$ and the spin quantization axis.

- If the degenerate states are near the Fermi level, the SOC-induced splitting may push some levels above $E_{F}$, lowering thus the energy [Daalderop et al. $(1990,1991,1994)$, Wang et al. (1993), Ravindran et al. (2001)].


## Contributions due to degenerate states: Important?



Gimbert \& Calmels PRB (2012)

- For extended systems, the degeneracy is limited only to a small part of the Brillouin zone, hence its influence on the MAE is limited.
[Lessard et al. (1997), Gimbert \& Calmels (2012)]


## Contributions due to degenerate states: Important?



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- For extended systems, the degeneracy is limited only to a small part of the Brillouin zone, hence its influence on the MAE is limited.
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- What if the $\mathbf{k}$-dependence is suppressed (as in the case of adatoms)?

Let us have a look...

## Calculate MAE for adatoms

$\mathrm{Fe}, \mathrm{Co}$, and Ni adatoms on $\mathrm{Au}(111)$ surface


- Fully relativistic Green's-function KKR formalism, solving the Dirac equation [Ebert, Ködderitzsch and Minár RPP (2011)]
- MAE evaluated via the torque
- Definition: $E_{\text {MAE }} \equiv E^{(x)}-E^{(z)}$ $E_{\text {MAE }}>0 \Leftrightarrow$ easy axis is normal to the surface
- Plain LDA, potential is subject to the ASA
- Adatoms geometry taken from VASP calculations


## Results for $E_{\text {MAE }}$ and magnetic moments

|  |  | $E_{\text {MAE }}$ <br> $(\mathrm{meV})$ | $\mu_{\text {spin }}^{(z)}$ <br> $\left(\mu_{\mathrm{B}}\right)$ | $\mu_{\text {orb }}^{(z)}$ <br> $\left(\mu_{\mathrm{B}}\right)$ | $\mu_{\text {orb }}^{(x)}$ <br> $\left(\mu_{\mathrm{B}}\right)$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Fe | adatom | 4.07 | 3.40 | 0.536 | 0.062 |
| Co | adatom | 4.42 | 2.13 | 0.218 | 0.206 |
| Ni | adatom | -1.63 | 0.67 | 0.063 | 0.158 |

Numbers may be interesting but they are not our focus.

Besides, the particular values could be affected by approximations we have made.
See, e.g., Khan et al. PRB 94144436 (2016) to learn more.

## DOS for $\mathrm{Fe}, \mathrm{Co}$ and Ni adatoms on $\mathrm{Au}(111)$





Magnetization perpendicular to the surface $\boldsymbol{M} \| \boldsymbol{z}$ (but it does not matter at this scale).

Let us focus on for the minority-spin states, where the most interesting stuff happens.

## What to do

## Plan：

Have a look at the spin－polarized orbitally－resolved DOS and monitor how it changes upon rotation of the magnetization．

Problem：
That＇s easier said than done．

## Spin-resolved and $m$-resolved DOS for $M \nVdash \hat{z}$

The way the DOS components $n_{\ell m}$ are defined depends on the reference frame.
Spherical harmonics $Y_{\ell m}$ can be defined in a global reference frame (fixed to the crystal lattice) or in a local reference frame chosen so that the $z^{(l o c)}$ axis is parallel to $\boldsymbol{M}$.

$$
\begin{aligned}
& n_{\ell m}^{(\mathrm{glo})}(E)=-\frac{1}{\pi} \Im\left\langle Y_{\ell m}^{(\mathrm{glo})}\right| G(E)\left|Y_{\ell m}^{(\mathrm{glo})}\right\rangle \\
& n_{\ell m}^{(\mathrm{loc})}(E)=-\frac{1}{\pi} \Im\left\langle Y_{\ell m}^{(\mathrm{loc})}\right| G(E)\left|Y_{\ell m}^{(\mathrm{loc})}\right\rangle
\end{aligned}
$$

(2) If $\boldsymbol{M} \nVdash \hat{\boldsymbol{z}}$, projecting the DOS in a global reference frame mixes the spin components because the spin quantization axis is no longer parallel to $\boldsymbol{M}$.

## Spin-resolved and m-resolved DOS for Co adatom



## Spin-resolved and m-resolved DOS for Co adatom






## Spin-resolved and m-resolved DOS for Co adatom

global frame:
"crystallographic"


global frame:
"crystallographic"


local frame:
"rotated"

If $\boldsymbol{M} \nVdash \hat{\boldsymbol{z}}$, then $n_{\ell m}^{(\mathrm{g} \mid 0) \uparrow}(E)$ and $n_{\ell m}^{(\mathrm{glo}) \downarrow}(E)$ are identical.

## How to maintain spin separation even if $M \nmid \hat{z}$

Resolving the spin can be done (with a good accuracy) once and for all, in any reference frame where it can be done.


- Start by resolving the DOS according to the spin in the local reference frame, where $\boldsymbol{M} \| \hat{\boldsymbol{z}}^{(\text {loc })}$ (without resolving the DOS into the $m$-components).
- Assume that the separation of spin components is maintained through subsequent transformation.
- All the further manipulations will be applied to minority-spin DOS and to majority-spin DOS separately.


## Transforming $n_{\ell m}$ between global and local frames

Transformation between $n_{L}^{(\text {loc })}$ and $n_{L}^{\text {(glo) }}$ generally not possible. Only the Green's function $G(E)$ can be properly transformed.

The transformation of the DOS can be done if $G(E)$ is diagonal in the global reference frame.
This is often the case.
Assume further:

$$
\begin{aligned}
n_{|m|}^{(\mathrm{glo})} & =n_{-|m|}^{(\mathrm{glo})} \\
n_{|m|}^{(\mathrm{loc})} & =n_{-|m|}^{(\mathrm{loc})}
\end{aligned}
$$



Then a tranformation between $n_{L}^{(\text {loc })}$ and $n_{L}^{(\text {glo })}$ can be performed.

## Change of DOS upon rotation of magnetization $M$







- The influence of the SOC is significantly larger for $\theta=0^{\circ}$ than for $\theta=90^{\circ}$.
- For $\theta=0^{\circ}$ the SOC splits the $m= \pm 2$ peak into two and shifts their positions in different directions.

Šipr et al. PRB 93, 174409 (2016)

Dependence of MAE on the position of the Fermi energy


- A sharp peak in the MAE at the energy where there is a peak for the $|m|=2$ component.
- For the Fe adatom this is overshadowed by the fact that, in this case, also the $|m|=1$ states are affected by SOC.


## DOS for inplane magnetization in a local frame



- The $\theta=90^{\circ}$ case cannot be directly compared the $\theta=0^{\circ}$ case because the definitions of the $m$-components differ.
- Effect of SOC for $\theta=90^{\circ}$ is less than for $\theta=0^{\circ}$.


## Model Hamiltonian

Simple model with only the crystal-field effects taken into account: Electron feels only the Coulombic field generated by charges located at the positions of the nuclei.

Restrict to $d$ electrons in an axial field (corresponding to $D_{4 d}$, i.e., antiprism symmetry).

$$
H=H^{(\mathrm{cry})}+H^{(\mathrm{ex})}+H^{(\mathrm{sOC})}
$$

- $H_{m s, m^{\prime} s^{\prime}}^{(\text {cry }}$ is determined by two parameters, resulting in three spin-degenerate energy levels.
- To distinguish between two orientations of the magnetization, we keep the spin quantization axis fixed (parallel to $z$ ) and vary the Hamiltonian $H^{(e x)}$.
- $\boldsymbol{H}^{(\mathrm{SOC})}=\xi \boldsymbol{L} \cdot \boldsymbol{S}$


## Energy levels for model Hamiltonian

Dependence of eigen-energies of the model Hamiltonian on the SOC strength $\xi$ for two orientations of $M$.






Thin dashed lines mark $\xi$ values appropriate for each element.

## Conclusions

- The effect of SOC on adatoms that only weakly hybridize with a substrate consists in splitting atomic-like levels that would be degenerate otherwise.
- The splitting is much larger if the magnetization is oriented perpendicular to the surface than if it is oriented parallel to the surface.
- The splitting is a combined result of crystal field, exchange splitting and spin-orbit coupling.
- If the originally degenerate level is close to the Fermi level, one of the peaks can be pushed above it, decreasing thereby the energy of the system.
- This results in a significant contribution to the magnetocrystalline anisotropy of adatoms.

